A New Phase Linking Algorithm for Multi-temporal InSAR based on the Maximum Likelihood Estimator

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1. INTRODUCTION

Multi-temporal InSAR techniques have gained more attention as satellite SAR images allow for the monitoring of large deforming areas with sub-centimeters accuracy. Since InSAR has been developed based on how the signal decorrelation over time can be accounted for, backscattering properties analysis is necessary. In the literature, two kinds of scatterers, Permanent Scatterer (PS) and Distributed Scatterer (DS) are distinguished. The current state-of-the-art multi-temporal InSAR approaches rely on 1) processing with point-wise time coherent PS, namely PS Interferometry [1], 2) the construction of redundant interferogram networks in DS Interferometry [2]; 3) the combination of PSI and DSI [3]. PSI approaches have widely been deployed for urban area monitoring, but its application to natural areas is often limited due to the weak PS points density. In DSI, Small BAseline Subset (SBAS) approaches use only small temporal and spatial baselines SAR image pairs in the interferogram network, with the objective to minimize signal decorrelation. Another important DSI approach corresponds to the Phase Linking (PL) approach. The main idea of this approach is to use all the $N \times (N-1)/2$ interferograms generated from a time series of N SAR images to yield the best estimate of $N-1$ single referenced phase difference [4]. This approach allows for a full exploitation of all possible combinations of a SAR image stack by formally taking the impact of the temporal decorrelation into account. Most recent advances in multi-temporal InSAR, e.g. EMI [5] and sequential estimator [6], have been developed from this baseline approach.

This paper aims to present a new algorithm for improving the estimation of interferometric SAR (InSAR) phases in the context of time series and phase linking (PL) approach. Based on maximum likelihood estimator (MLE) of a multivariate Gaussian model, the estimation of the InSAR phases is solved using the Block Coordinate Descent algorithm. Compared to the state-of-the-art PL approaches [4, 3, 7], the main improvement of our MLE based PL approach (namely MLE-PL for the sake of brevity) lies on the joint estimation of the coherence and the InSAR phases instead of the use of a plug-in coherence estimate obtained from the sample covariance of the data or the modeling of the temporal decorrelation of the target under observation. Results of synthetic simulations confirm the improvement brought by the proposed estimator.

2. MODEL AND METHODOLOGY

A time series of N SAR images are stacked along the temporal and spatial dimensions into a cube. Sliding window $\mathbf{x}_i$ contains a local observation for N snapshots: $\mathbf{x}_i = [x_i^0, ..., x_i^{N-1}]^\top$

Within the scope of this study, we assume that the set $\{\mathbf{x}_i\}_{i=1}^L$ with $\mathbf{x}_i \in \mathbb{C}^N$, $\forall i \in [1, L]$ are distributed scatterers which are spatially homogeneous over L adjacent pixels. $\{\mathbf{x}_i\}_{i=1}^L$ is thus a set of independent and identically distributed (i.i.d.) vectors. As in the current literature, we assume that $\mathbf{x}_i$ follows a zero-mean complex circular Gaussian distribution with the probability density function (PDF) of

$$f(\mathbf{x}, \mathbf{C}) = \frac{1}{\pi^N \text{Det}(\mathbf{C})} \exp\left(-\mathbf{x}^H \mathbf{C}^{-1} \mathbf{x}\right) \tag{1}$$

The second moment of $\mathbf{x}$ relates to interferograms can be rewritten in matrix form as

$$\mathbb{E}[\mathbf{x}\mathbf{x}^H] \triangleq \mathbf{C} = \text{ediag}(\boldsymbol{\theta}) \underbrace{((\boldsymbol{\sigma}\boldsymbol{\sigma}^\top) \circ \boldsymbol{\Gamma})}_{\boldsymbol{\Sigma}} \text{ediag}(\boldsymbol{\theta})^H \text{, with } \text{ediag}(\boldsymbol{\theta}) = \begin{pmatrix} \exp(j\theta_0) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \exp(j\theta_{N-1}) \end{pmatrix} \tag{2}$$

The matrix $\mathbf{C}$ fully characterises the InSAR phases and coherence. The InSAR principle is then to estimate all the elements of $\mathbf{C}$. Since $\boldsymbol{\Sigma}$ is unknown in practice, several papers [4, 3, 7] used a plug-in estimate, the sample covariance matrix (SCM) $|\mathbf{S}|$ (entry-wise modulus) was used as an estimate of $\boldsymbol{\Sigma}$, yielding a two-step algorithm. This approach yields good results in practice, but is known to be sub-optimal since $|\mathbf{S}|$ is a biased estimate of $\boldsymbol{\Sigma}$. We propose to jointly estimate the matrix $\mathbf{C}$ and the phase difference by solving the MLE with an iterative approach.

We reparameterize the equation (2) as

$$\mathbf{C}(\boldsymbol{\Sigma}, \boldsymbol{\theta}) = \text{ediag}(\boldsymbol{\theta})\boldsymbol{\Sigma}\, \text{ediag}(\boldsymbol{\theta})^H \tag{3}$$

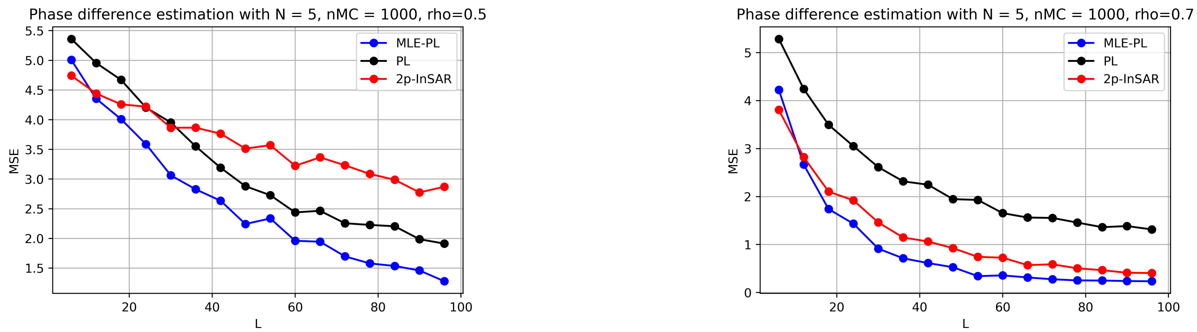

Fig. 1. MSE of InSAR estimates using PS-like, PL and MLE-PL with Gaussian distributed input data. $N = 5$, 1000 Monte Carlo trials.

The maximum likelihood estimator corresponds to the solutions of the (negative log likelihood) minimization problem

$$\begin{array}{ll} \underset{\boldsymbol{\Sigma},\boldsymbol{\theta}}{\text{minimize}} & \mathcal{L}\left(\mathbf{C}\left(\boldsymbol{\Sigma},\boldsymbol{\theta}\right)\right) \\ \text{subject to} & \boldsymbol{\Sigma} \text{ real symmetric} \\ & \theta_0 = 0 \end{array} \tag{4}$$

where $\mathcal{L}$ is the log-likelihood associated with model (1). We propose a Block Coordinate Descent (BCD) algorithm to compute the MLE. This corresponds to an algorithm that iteratively minimizes the objective w.r.t to each variable ($\boldsymbol{\Sigma}$ or $\boldsymbol{\theta}$) while keeping the other fixed.

With fixed $\boldsymbol{\theta}$, $\boldsymbol{\Sigma}$ can be obtained as the real part of the modified sample covariance matrix

$$\boldsymbol{\Sigma}^{\star} = \text{real}(\text{ediag}(\boldsymbol{\theta})^H \mathbf{S} \text{ediag}(\boldsymbol{\theta})) \tag{5}$$

By fixing $\boldsymbol{\Sigma}$, the problem is referred to as phase-linking [7] or phase triangulation [4], which reads

$$\begin{array}{ll} \underset{\boldsymbol{\theta}}{\text{minimize}} & \mathbf{w}_{\boldsymbol{\theta}}^H (\boldsymbol{\Sigma}^{-1} \circ \mathbf{S}) \mathbf{w}_{\boldsymbol{\theta}} \\ \text{subject to} & \theta_0 = 0 \end{array}, \text{ with } \mathbf{w}_{\boldsymbol{\theta}} = \left[e^{j\theta_0}, \ \cdots, \ e^{j\theta_{N-1}} \right]^{\top} \tag{6}$$

$\boldsymbol{\theta}$ is solved using Majorization-Minimization (MM) algorithm.

3. SIMULATION

To assess the performance of the proposed algorithm, we choose to generate a matrix $\mathbf{C}$ from (2). The correlation matrix $\boldsymbol{\Sigma}$ is chosen as a Toeplitz matrix with correlation coefficient $\rho = 0.5$ and 0.7. The N simulated InSAR phases are random values from $(-\pi, \pi)$. At last, L i.i.d samples are then simulated from distribution (1). Spatial window of L pixels is ranging from 6 to 100 with $N = 5$ dates and 1000 number of Monte Carlo simulations. For comparison, other methods are also tested. 2p-InSAR is a common InSAR processing in which InSAR phases are only estimated in spatial dimension and with the use of a multilooking window. PL is the conventional phase linking [7] which is a 2-step approach where the true coherence $\boldsymbol{\Sigma}$ is replaced by $|\mathbf{S}|$. In the following part of this study, the number of iterations is set to 10 to ensure the convergence of the MLE-PL.

Figure 1 presents the Mean Squared Error (MSE) of estimated InSAR phases for the different methods and in terms of the number of pixels within a spatial window of L pixels. For all the considered correlations, MSE for all the methods decrease with the number of available samples which is expected. PL and MLE-PL show lower MSE for medium and low correlations compared to PS-like. In general, MLE-PL needs more samples to achieve its optimal MSE compared to PL. This is explained by the estimation of both InSAR phases and coherence in MLE-PL.

4. REFERENCES

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