Fast Computation of the Optimal ISL Filter for Large-Scaled Problems

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The continuous growth of communication applications gradually congests the electromagnetic spectrum. Concurrently, the need of alleviating the bandwidth occupation, leads to combine two different applications, such as radar and communications systems [1]. One possible way is to design a joint radar communication framework that exploits the same signal and allows simultaneous functioning at the same frequency bandwidth.

We consider here the context of a joint synthetic aperture radar (SAR) and communication system. A good quality SAR image is usually obtained by transmitting a chirp waveform and processing the received signal with a matched filter [2]. However, such a signal cannot transmit information, so communication codes will be considered here, in order to perform both applications simultaneously, with different transmitting codes at each pulse to maximize the data rate. Unfortunately, such communication codes lead to potentially strong sidelobes after matched filtering which may deteriorate the image contrast. Thus, a different compression filter, called mismatched filter [3], must be used in order to mitigate the sidelobe level. Here, we will consider a mismatched filter minimizing the Integrated Sidelobe Level (ISL) criterion in order to improve the image contrast.

Let $\mathbf{s} \in \mathbb{C}^N$ be a sampled signal and $\mathbf{q} \in \mathbb{C}^K$ with $K \geq N$ a mismatched filter, then the optimization problem to find the optimal $\mathbf{q}$ that minimizes the integrated sidelobe level (*ISL*) is expressed as an L_2 norm problem [4],

$$\begin{aligned} \min_{\mathbf{q}} \quad & \mathbf{q}^H \mathbf{M} \mathbf{q}, \\ \text{s.t.} \quad & \mathbf{s}^H \mathbf{q} = \mathbf{s}^H \mathbf{s}, \\ & \mathbf{q}^H \mathbf{q} \leq \alpha \mathbf{s}^H \mathbf{s}, \end{aligned} \tag{1}$$

where $\mathbf{M}$ is a symmetric semi-definite positive matrix defined in [5]. As mentioned in [4], both the cost function and the constraints are convex, so the problem can be solved using a convex solver, such as the *CVX: Matlab Software for Disciplined Convex Programming* [6]. However, *CVX* is not suitable for large-scale problems: in our case, due to the signal oversampling and the large number of different transmitted waveforms, the use of CVX would lead to completely prohibitive computational cost. Hence the motivation to solve the problem (1) via a much faster algorithm.

The proposed method provides the optimal filter by solving the dual problem. Whereas the primal problem solved by the convex method requires a numerical interior-point algorithm over thousands of variables, it is possible to show that most variables involved in the dual problem can be determined analytically, while only one variable must be obtained numerically, thus speeding up a lot the computation. Besides it can be shown here that the solution of the dual problem coincides with the optimal solution of the primal problem. The proposed procedure is the following:

1. The dual function needs to be computed, which is defined as the minimum of the Lagrangian function.
2. The Lagrangian function is computed, along with its gradient with respect to $\mathbf{q}$, in order to explicitly express the dual function,
$$g(\lambda, \nu) = -\lambda \alpha \mathbf{s}^H \mathbf{s} - \nu \mathbf{s}^H \mathbf{s} - \frac{1}{4} \nu^2 \mathbf{s}^H (\mathbf{M} + \lambda \mathbf{I})^{-1} \mathbf{s}. \tag{2}$$
3. In order to maximize the concave dual function g, its two variables ν and λ need to be determined.
4. ν is determined analytically and λ numerically, through the Newton-Raphson algorithm.
5. Once the optimal pair (λ^*, ν^*) is determined, the optimal filter of the dual space is computed,
$$\mathbf{q}^* = -\frac{1}{2} \nu^* (\mathbf{M} + \lambda^* \mathbf{I})^{-1} \mathbf{s}. \tag{3}$$

We propose to apply this method to specific communication codes, called continuous phase frequency-shift keying (CPFSK) codes [7]. 6522 different CPFSK signals are considered here, each composed of 3000 samples. The length of the associated mismatched filter is set to 9000 samples. Table 1 shows the computational time needed for the proposed method and the Matlab *CVX* solver for one and for all the mismatched filters. It appears that our method computes the filters almost 250 times faster than the *CVX* solver, while providing the same solution as CVX.

Once all the filters are generated, re-synthesized SAR images are created. More precisely, a real data image acquired with chirp signals is considered from which the scene reflectivity is obtained by deconvolving the image with the impulse response of the chirp signal. It is then possible to convolve this scene reflectivity with each of the CPFSK signals, and perform the matched or the mismatched filter to obtain synthesized outputs.