On the false alarm probability of the Normalized Matched Filter for off-grid target detection

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The goal of radar systems is to detect the presence of targets with unknown parameters in unknown environment. Classically, in presence of unknown parameters, the commonly used detection strategy is the Generalized Likelihood Ratio test (GLRT) that replaces the unknown parameters by their Maximum Likelihood estimates (MLE) in the detection test. Unfortunately, analytical MLE solutions are not available for some target parameters of interest (Doppler shift, distance, and direction for example).

Therefore, for ease of implementation, most detection strategies assume that target parameters lie over a discrete set, called the grid. However, target parameters have no reason to fall exactly on the grid, since they are distributed over a continuous range. This mismatch between the tested parameters and the real target parameters deteriorates the response of most state-of-the-art detection tests.

For example, it was shown [2] that the Normalized Matched Filter (NMF) detector used in non-Gaussian contexts is very sensitive to steering vector mismatch, potentially leading to a dramatic deterioration of the detection performance: in particular for some mismatch values, it was shown that the asymptotic detection probability (P_D) tends to 0 at high SNR: this phenomenon occurs [3] for Probabilities of False Alarm (P_{FA}) as high as 10^{-3}. This can be seen in Figure 1: this response falls below the detection thresholds at the edge of the cell.

This advocates for the use of the true GLRT which considers the target parameters as unknown. Besides its lack of known closed form, one of the issue with this detector is that its analytical P_{Fa} −threshold relationship is unknown. In our work, we derive such a relationship with a geometrical approach.

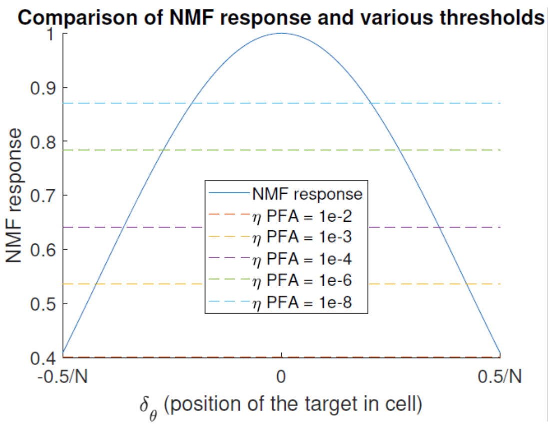

Figure 1 *Comparison of the noiseless NMF response with various thresholds η under white noise for a steering vector of size N=10*

Note that the NMF is an angle detector: it will detect any signal that falls into a cone centered on the expected signal with an angle depending on the threshold. Computing the P_{FA} of the NMF geometrically then reduces to computing the ratio between the surface of the cone projected on the unit sphere (a spherical cap) and the surface of the sphere, as illustrated on Figure 2.

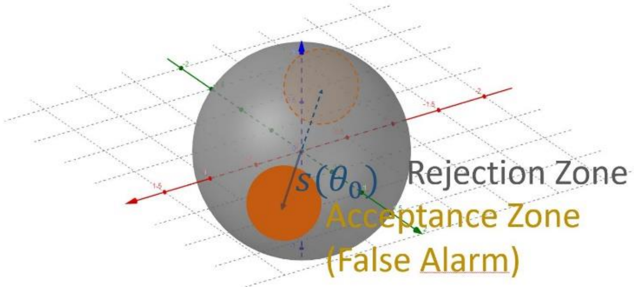

Figure 2 *Acceptance and Rejection zones for the NMF with reference signal* $\boldsymbol{s}(\theta_0)$.

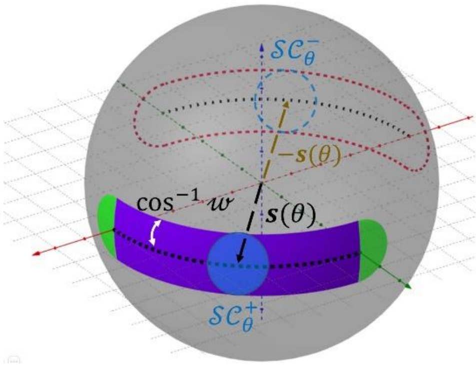

Figure 3 *Tube in purple and dashed around the manifold spanned by the steering vectors of the cell. It is the union of the spherical caps* $\mathcal{SC}_\theta$ *for each parameter in the cell.*

A signal falls in the acceptance zone of the GLRT if it falls in any of the spherical caps around tested signals. Thus the signal will be accepted if it falls in a tube around the manifold spanned by the steering vectors in the cell under test, as can be seen on Figure 3 for signals in R^3. What we have to do in order to find the P_{FA}-threshold relationship, then, is to compute the ratio between the surface of the tube and the surface of the unit sphere. Hotelling [1] gives a formula to compute the surface of a tube around a curve in R^N, but the Radar signals under consideration are complex. In our work, we show that this problem is equivalent to computing the surface of a tube around a 2D manifold. Thanks to a result from [4], we are able to compute this surface when the threshold is low enough to avoid overlapping issues.

This enables us to compute an analytical P_{FA}-threshold relationship. In the Figure 4, we test our formula numerically and show that it fits the simulation perfectly except for high P_{FA} where the tube overlaps (it draws back into itself) and the formula becomes an upper bound.

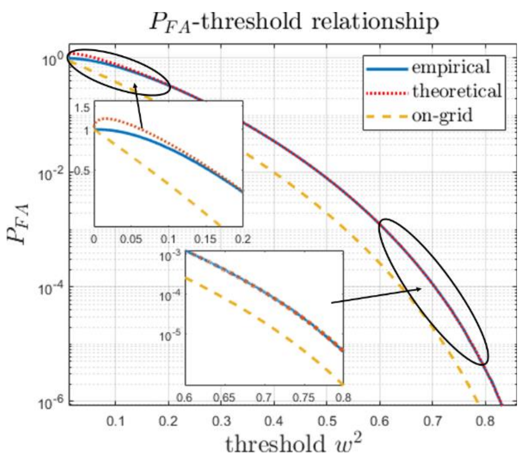

Figure 4 *Comparison between simulated threshold and simulated ones, with the on-grid relationship as a reference.*

[1] Hotelling, H. (1939). Tubes and spheres in n-spaces, and a class of statistical problems. American Journal of Mathematics, 61(2), 440-460.

[2] Rabaste, O., & Trouve, N. (2014). Geometrical design of radar detectors in moderately impulsive noise. IEEE Transactions on Aerospace and Electronic Systems, 50(3), 1938-1954.

[3] Rabaste, O., Bosse, J., & Ovarlez, J.-P. (2016). Off-grid target detection with normalized matched subspace filter. In 2016 24th European Signal Processing Conference (EUSIPCO) (pp. 1926-1930). IEEE.

[4] Johnstone, I., & Siegmund, D. (1989). On Hotelling's formula for the volume of tubes and Naiman's inequality. The Annals of Statistics, 184-194.