5th SONDRA Workshop

Avignon, 14-16 June 2022

Joint estimation of location and positive-definite scatter matrix in elliptical distributions

Stefano Fortunati[1,2], Alexandre Renaux[1] and Frédéric Pascal[1]

[1]Université Paris-Saclay, CNRS, CentraleSupélec, Laboratoire des signaux et systèmes, 91190, Gif-sur-Yvette, France,
[2]DR2I-IPSA, 94200, Ivry sur Seine, France.

(Oral presentation)

Abstract
Inferring the correlation structure of a set of centered data if a key step in many signal processing and machine learning procedures. Among others, radar/sonar detection, image segmentation, dimension reduction, distance learning and clustering rely on the estimation of the covariance/correlation matrix of an acquired data set [1]. Along with the need of an estimation of the covariance matrix, there is another common aspect in all the above-mentioned applications: the non-Gaussian and heavy-tailed nature of the data. As a consequence, popular Gaussian and pseudo-Gaussian inference procedures may present a dramatic performance decay as extensively shown in statistics and signal processing literature (see e.g. [2] and the references therein).

Motivated by a wide range of experimental evidences and measurement campaigns, the (Real or Complex) Elliptically Symmetric (ES) model has been recently adopted to characterize the statistical data behavior [3]. Together with its generality, the second feature that has placed the elliptical model in the spotlight of signal processing and machine learning communities is its "parsimony" in terms of required parameters. In fact, CES model is fully specified by two *finite dimensional* parameters, i.e. a location vector and a positive-definite covariance/scatter matrix (as the classical Gaussian model) and by an *infinite dimensional* functional [4,5] parameter, usually called *density generator*, characterizing the data heavy-tailedness.

The aim of this work is then to investigate the joint estimation of the location vector and the shape matrix of a set of independent and identically Complex Elliptically Symmetric (CES) distributed observations from both the theoretical and computational viewpoints. This joint estimation problem is framed in the original context of semiparametric models allowing us to handle the (generally unknown) density generator as an *infinite-dimensional* nuisance parameter. In the first part of the paper, a computationally efficient and memory saving implementation of the robust and semiparmaetric efficient R-estimator for shape matrices is derived. Building upon this result, in the second part, a joint estimator, relying on the Tyler's M-estimator of location and on the R-estimator of shape matrix, is proposed and its Mean Squared Error (MSE) performance compared with the Semiparametric Cramér-Rao Bound (CSCRB) [6,7].

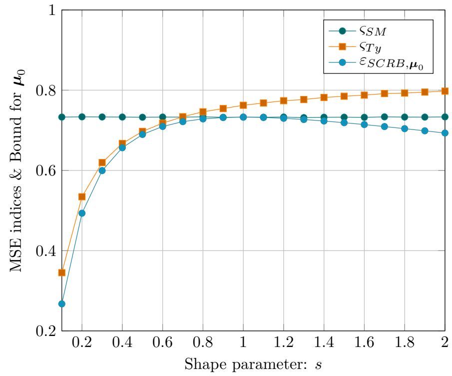

Figure 1: MSE in the estimation of the location vector $\boldsymbol{\mu}_0$ for the sample mean and for the Tyler's estimator.

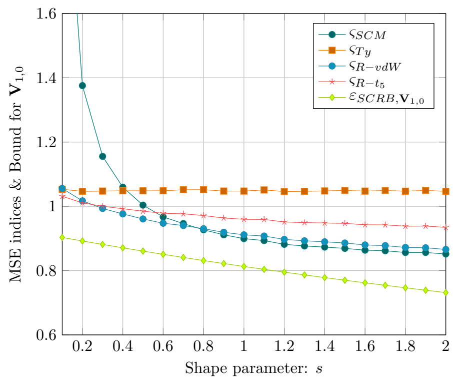

Figure 2: MSE in the estimation of the scatter $\mathbf{V}_{1,0}$ for the SCM, the Tyler's estimator and the R-estimator exploiting both the van der Waerden and the t_5-score functions..

[1] C. M. Bishop, *Pattern Recognition and Machine Learning (Information Science and Statistics)*. Springer, 1 ed., 2007.

[2] A. M. Zoubir, V. Koivunen, E. Ollila, and M. Muma, *Robust Statistics for Signal Processing*. Cambridge University Press, 2018.

[3] E. Ollila, D. E. Tyler, V. Koivunen, and H. V. Poor, "Complex elliptically symmetric distributions: Survey, new results and applications," *IEEE Transactions on Signal Processing*, vol. 60, no. 11, pp. 5597–5625, 2012.

[4] P. Bickel, C. Klaassen, Y. Ritov, and J. Wellner, *Efficient and Adaptive Estimation for Semiparametric Models*. Johns Hopkins University Press, 1993.

[5] S. Fortunati, A. Renaux, and F. Pascal, "Robust semiparametric efficient estimators in complex elliptically symmetric distributions," *IEEE Transactions on Signal Processing*, vol. 68, pp. 5003–5015, 2020.

[6] S. Fortunati, F. Gini, M. S. Greco, A. M. Zoubir, and M. Rangaswamy, "Semiparametric inference and lower bounds for real elliptically symmetric distributions," *IEEE Transactions on Signal Processing*, vol. 67, pp. 164–177, Jan 2019.

[7] S. Fortunati, F. Gini, M. S. Greco, A. M. Zoubir, and M. Rangaswamy, "Semiparametric CRB and Slepian-Bangs formulas for complex elliptically symmetric distributions," *IEEE Transactions on Signal Processing*, vol. 67, pp. 5352–5364, Oct 2019.