

Multidimensional stationary time series Machine Learning for Radar Clutter Classification

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Abstract—We present a method to classify complex valued stationary centered Gaussian autoregressive time series. Our initial motivation comes from radar signal processing and especially radar clutter classification, which we detail in Section I. This issue has already been addressed in previous works, in particular by Frédéric Barbaresco [4], [7], [16], [19], [20], Le Yang [2], [28], Alice Le Brigant [9] and Alexis Decurninge [1], [13].

Keywords: Complex stationary centered Gaussian autoregressive time series, multidimensional time series, machine learning, Information geometry, Riemannian manifold, Block-Toeplitz matrices, Siegel disk, radar clutter, spatio-temporal correlation.

I. RADAR MOTIVATIONS

The initial motivation for studying complex valued stationary centered Gaussian autoregressive time series comes from radar signal processing, in particular the study of radar clutter. In radar semantics, we distinguish between moving objects of primary interest which we call targets and information related to the radar environment which we call clutter. Radar clutter is therefore the information recorded by a radar related to seas, forests, fields, cities and other environmental elements that surround the radar. In order to better distinguish targets from clutter, it may be interesting to develop machine learning algorithms to recognize different types of clutter. Knowledge of radar clutter can be used to obtain a constant false alarm rate (CFAR) detection estimator [10], [13], [26].

To study the characteristics of the complex valued time series associated with radar clutter, it is common to assume that they are stationary centered autoregressive Gaussian time series [8]. The assumption of stationarity of the time series is here justified by extremely short observation times of the same zone of the environment. The laws of these time series are represented in Riemannian manifolds in the works of Le Yang [2], [28] and Alice Le Brigant [9]. This representation model is briefly summarized in Section II; it is applied to radar clutter clustering in the thesis work of Yann Cabanes [10].

In order to refine the study of the characteristics of radar clutter, we want to add spatial information to the temporal information contained in each time series by studying the

correlation between time series recorded in spatially close cells. For this, we have developed a spatio-temporal model [10]. In order to provide the space of the coefficients of this spatio-temporal model with a Riemannian metric, the more general case of multidimensional stationary time series is studied in [10] and briefly summarized in Section III.

II. ONE-DIMENSIONAL STATIONARY TIMES SERIES

The study of one-dimensional stationary radar time series was carried out by Frédéric Barbaresco in [3]–[7], [21]. In these works, the stationary radar time series are represented in the product space $\mathbb{R}_+^* \times \mathcal{D}^{n-1}$ where \mathcal{D} represents the complex unit disk. This space is endowed with a Riemannian metric inspired by information geometry. The space \mathbb{R}_+^* is used to represent the average quadratic power of the studied time series. The product space \mathcal{D}^{n-1} represents the coefficients of the autoregressive model, it therefore represents the Doppler information contained in the time series.

The Burg algorithm is used to estimate the coefficients of the autoregressive model from a recorded time series. This algorithm is presented in the works of Frédéric Barbaresco and Alexis Decurninge [1], [13].

The Riemannian metric constructed on the space $\mathbb{R}_+^* \times \mathcal{D}^{n-1}$ is presented by Frédéric Barbaresco in [19], [22], [17], [16], [15] and related works [12], [25], [14]. This metric is also detailed in the thesis works of Le Yang [2], [28], Alice Le Brigant [9] and Yann Cabanes [10]. We refer to Shun-ichi Amari’s book [27] for a full presentation of the information geometry tools used to construct this metric. We denote $\mathbb{R}^{++} \times \mathbb{D}^{n-1}$ the Riemannian manifold presented in these works: the manifold $\mathbb{R}^{++} \times \mathbb{D}^{n-1}$ corresponds to the space $\mathbb{R}_+^* \times \mathcal{D}^{n-1}$ endowed with a Riemannian metric inspired by information geometry. The computation of the mean and median in this manifold is used to detect radar targets in the work of Le Yang [2], [28]. The study of the curves of the manifold $\mathbb{R}^{++} \times \mathbb{D}^{n-1}$ is applied to the recognition of radar targets in the work of Alice Le Brigant [9]. In the thesis work of Yann Cabanes [10],

the manifold $\mathbb{R}^{++} \times \mathbb{D}^{n-1}$ is used for radar clutter clustering and more generally to the classification of complex stationary centered Gaussian autoregressive time series.

III. MULTIDIMENSIONAL STATIONARY TIMES SERIES

Complex multidimensional stationary autoregressive centered Gaussian time series are represented in a Riemannian manifold in the thesis work of Yann Cabanes [10]. As in the case of one-dimensional time series, it is possible to represent multidimensional time series by the coefficients of the autoregressive model. In the case of multidimensional time series, these autoregressive coefficients are square matrices. In the article written by Ben Jeuris and Raf Vandebril [23], the matrix coefficients of the autoregressive model are slightly modified to belong to the Siegel disk \mathbb{SD}_N (set of complex matrices $N \times N$ of singular values strictly less than 1). The multidimensional stationary time series can then be represented in the space $\mathcal{H}_N^+ \times \mathcal{SD}_N^{n-1}$, where \mathcal{H}_N^+ is the space of Hermitian Positive Definite (HPD) matrices. This space can be endowed with a Riemannian product metric, the construction of which is detailed in the article written by Ben Jeuris and Raf Vandebril [23]. The product metric on the space $\mathcal{H}_N^+ \times \mathcal{SD}_N^{n-1}$ induces a Riemannian metric on the spaces \mathcal{H}_N^+ and \mathcal{SD}_N . The metric of the Siegel disk \mathcal{SD}_N have been studied by Frédéric Barbaresco in [20], [18] and the related work [24]. The Riemannian logarithm map, the Riemannian exponential map and the sectional curvature of the Riemannian manifold defined on the Siegel space \mathcal{SD}_N have been given by Yann Cabanes in [10], [11]. These geometric tools are essential for the use of certain machine learning algorithms, in particular algorithms involving a computation of the mean as the k-means algorithm.

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