Robust PCA for Through-the-wall radar imaging

Through-the-wall radar imaging (TWRI) is an ongoing field of research which aims at investigating the inside of a building from its outside. In the most common setting, it seeks to detect or monitor stationary targets. Departing from usual delay-and-sum techniques, sparse recovery problems have been proposed to solve this detection problem. These methods rely on a preprocessing step where an appropriate separation of wall and target subspaces is first performed to remove the front wall response. In this work, we explore one-step methods using joint low-rank and sparse decomposition methods through the Robust PCA (RPCA) framework. The novelty is a one-step recovery from a structured inversion problem for which we tailor an Alternating Direction Method of Multipliers (ADMM) algorithm.

In our work, we describe the received SAR signal, for the mth frequency and nth position, as [Moeness13]:

$$\begin{aligned} y(m,n) &= \sigma_w \exp\left(-j\omega_m \tau_w\right) \\ &+ \sum_{i=0}^{R-1} \sum_{p=0}^{P-1} \sigma_p^{(i)} \exp(-j\omega_m \tau_{p,n}^{(i)}) \end{aligned} \tag{1}$$

Where ω_m is the m^{th} angular frequency, $\sigma^{(i)}_p$ and $\tau^{(i)}_{p,n}$ are the attenuation coefficient and time delay from the nth SAR position to the pth target through the ith multipath (where the first one is the direct path). This leads us to the following expression:

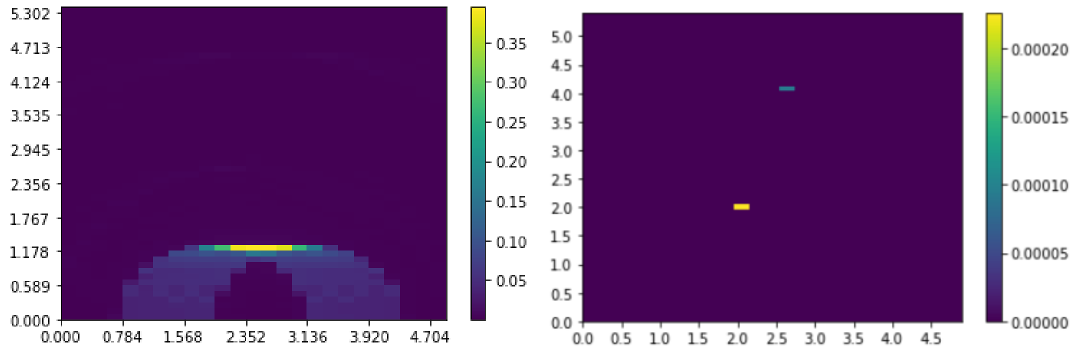
$$\begin{aligned} \underbrace{[\mathbf{y}_0|\dots|\mathbf{y}_{N-1}]}_{=\mathbf{Y}} &= \underbrace{[\mathbf{l}|\dots|\mathbf{l}]}_{=\mathbf{L}} + \\ & \underbrace{[\mathbf{\Psi}_0|\dots|\mathbf{\Psi}_{N-1}]}_{=\mathbf{\Psi}} \underbrace{\begin{bmatrix} \mathbf{r} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{r} \end{bmatrix}}_{=\mathbf{S}} \\ \implies \mathbf{Y} &= \mathbf{L} + \mathbf{\Psi}\mathbf{S} = \mathbf{L} + \mathbf{\Psi}\left(\mathbf{I}_N \otimes \mathbf{r}\right) \end{aligned} \tag{6}$$

where $\mathbf{Y} \in \mathbb{C}^{M\times N}$ is the data matrix, $\mathbf{L} \in \mathbb{C}^{M\times N}$ is a low-rank matrix of front wall returns, $\mathbf{\Psi} \in \mathbb{C}^{M\times N_x N_z RN}$ is a dictionary mapping to the target returns and $\mathbf{S} \in \mathbb{C}^{N_x N_z RN\times N}$ is the associated sparse matrix containing the scene vector.

This is a formulation that fits RPCA 'with dictionary' [Mardani13] as seen in (7). However, the structure of the sparse matrix (see (6)) causes problems of convergence to this algorithm. We therefore proposed an alternative problem (8) and developed an algorithmic resolution (Algorithm 1) through ADMM [Boyd11] which uses the Augmented Lagrangian (ALM) form. On simulated data, we recovered correctly the position of 2 targets in an empty room where the traditional back projection will fail due to the presence of the wall.

$$\begin{aligned} \min_{\mathbf{L},\mathbf{S}} \quad & \|\mathbf{L}\|_* + \lambda\|\mathbf{S}\|_1 \\ \text{s.t.} \quad & \mathbf{Y} = \mathbf{L} + \mathbf{\Psi}\mathbf{S} \end{aligned} \tag{7}$$

$$\begin{aligned} \min_{\mathbf{L},\mathbf{r}} \quad & \|\mathbf{L}\|_* + \lambda\|\text{unvec}(\mathbf{r})\|_{2,1} \\ \text{s.t.} \quad & \mathbf{Y} = \mathbf{L} + \mathbf{\Psi}\left(\mathbf{I}_N \otimes \mathbf{r}\right) \end{aligned} \tag{8}$$



Result of backpropagation(left) and our method (right)

We will extend the cost function to a robust data fitting term through the Huber function. This should make our method more performant in degraded settings: non-straight SAR displacement from manipulation mishandling, etc. We will work on more complex simulated data based on Finite-difference time-domain (FDTD) methods. Ultimately, we will create our own data from real-world experiments. Those methods will allow us to consider the impact of clutter and more complex scenes on algorithms.

References

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[Boyd11] S. Boyd, N. Parikh, E. Chu, B. Peleato and J. Eckstein (2011), "Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers", Foundations and Trends® in Machine Learning: Vol. 3: No. 1, pp 1-122.