Quantum computing - Phase-Coded Radar Waveform AI-Based Augmented Engineering and Optimal Design by Quantum Annealing

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In this work, we explore the resolution of the ISLR problem in the context of quantum computing. The Integrated SideLobe Ratio (ISRL) problem is computationally complex, and we cannot solve it exactly for large instances. In the other hand, quantum computing is a promising field that aims to use the particularities of quantum mechanics to speed up some heavy computational tasks.

The ISLR problem consists in finding optimal sequences of phase shifts in order to minimize the mean squared cross-correlation sidelobes of a transmitted radar signal and a mismatched replica. Given a finite set of quantized phase shifts, the exponentially increasing number of possible sequences leads to long-time processing and approximated results. Currently, ISLR does not seem to be easier than the general polynomial unconstrained binary problem, which is NP-hard.

Quantum computing consists in harnessing the physical properties of quantum mechanics, such as entanglement or superposition, to perform computations more efficiently than any classical computer. Currently, there exists no large-scale reliable quantum processor. However, we can already explore the capability of quantum devices by using existing small noisy devices, or simulations. The two candidates for near-term quantum computing are Adiabatic Quantum Computers (AQC) and Noisy Intermediate-Scale Quantum (NISQ) computers. AQC are analogical devices that specifically solve problems known as Quadratic Unconstrained Binary Optimization (QUBO). NISQ computers are logical computers that use gates and instructions, but are limited to low-depth algorithms because of noise and instability issues. For now, the question of which approach will provide the best performances is still open. In both cases, the problem involves a Hamiltonian, and the goal is to find the binary sequences minimizing its value.

In a first part, we study the problem on an AQC, designed to solve problems in the form of a QUBO. Due to the non-quadratic form of the ISLR problem, the related Hamiltonian requires a reformulation, which increases the size of the search space and leads to irrelevant solutions. Consequently, additional penalty Hamiltonians have to be implemented to reduce the time required to get actual solutions. We also introduce an additional non-quadratic constraint to ensure the obtention of actual solutions, however, we have not yet found any way to enforce it. For the results, we ran our code on D-Wave Advantage AQC through the Leap service and compared performances with and without penalty Hamiltonians, respectively called 2-constraint model and unconstrained model in the figure below, enlightening the added value of quadratic constraints.

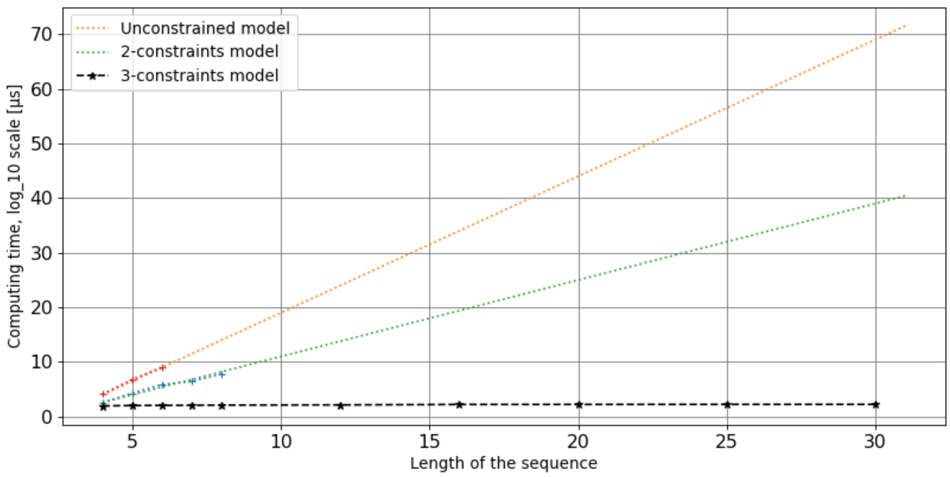

Fig. 1. Graphical representation of computing time to obtain a solution. 3-constraints model represents hypothetic results obtained with the non-quadratic constraint implemented

In a second part, we show how one can solve the ISLR problem using a gate-based quantum computer. For this part, we implement the Quantum Approximate Optimization Algorithm (QAOA), an algorithm designed to solve optimization problems on NISQ devices. For a length-5 signal, we provide the circuit that we ran on an IBM Falcon r4L quantum device. We also performed simulations for signals up to length 29.

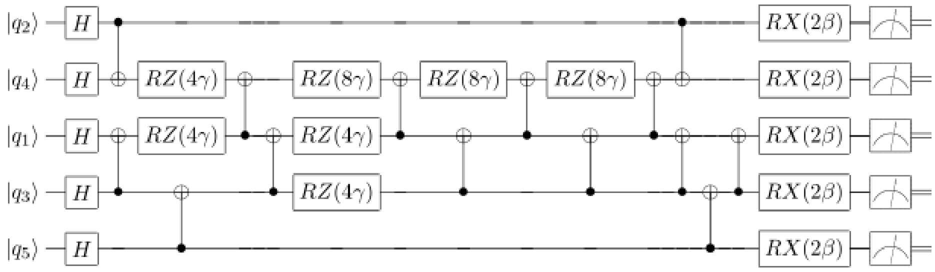

Fig. 2. Complete circuit of a depth-1 QAOA algorithm for a length-5 signal

In both cases, we study the scalability of our approach. More broadly, this study enlightens the limits and the potential of quantum computation in terms of speedup and high-scaled resolution of a class of combinatory optimization problems. Currently, only small devices are available, but we could manage to find the optimal solutions for the instance that fit on them. With the constant improvement of the quantum devices, we expect to be able to run larger instances in a near future. However, even with those experiments on small signals, we show that the topology and the connectivity of the hardware has a crucial importance for the scaling.

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